

MULTI-STRATEGY MULTI-OBJECTIVE PARTICLE SWARM OPTIMIZATION ALGORITHM BASED ON FRÉCHET DISTANCE AND VARIABLE NEIGHBORHOOD SEARCH

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In engineering fields such as workshop scheduling, Multi-objective optimization problems are highly challenging, as traditional algorithms tend to trap in local optima and struggle to balance solution set diversity and convergence. To address these limitations, this article proposes a particle swarm optimization algorithm combining multiple strategies (MS-MOPSO). MS-MOPSO constructs a co-evolutionary search framework integrating particle swarm optimization and grey wolf optimizer, incorporates a leading individual selection mechanism that combines average Fréchet distance-based curve similarity and Pareto dominance degree to guide evolutionary direction, and enhances local optimization capability through a variable neighborhood search with three structures. Performance evaluation against three comparative algorithms is conducted using the DTLZ1–5 multi-objective test suites, with significance testing further verifying the algorithm's performance. Results demonstrate that MS-MOPSO exhibits excellent solution set convergence, diversity and uniformity, effectively confirming its effectiveness and superiority in solving multi-objective optimization problems.

Keywords: multi-objective optimization; particle swarm optimization algorithm; Fréchet distance; variable neighborhood search

1. Introduction

To align with real-world production and life, multi-objective optimization has emerged as a research hotspot across fields. However, inherent conflicts between objectives make simultaneous optimization challenging—an issue that intensifies with increasing objective dimensions, often resulting in compromised non-inferior solutions. Thus, effectively addressing high-dimensional multi-objective optimization remains a critical and urgent task in the field.

Existing literature shows that multi-objective optimization is predominantly addressed via intelligent bionic algorithms, yet each algorithm has inherent limitations that hinder meeting efficiency and quality requirements. Consequently, fusing mechanisms of different algorithms for mutual complementarity has become

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a major research direction. For instance, Wang et al. [1] proposed a dual-swarm genetic algorithm integrating forward-backward scheduling and schedule-generated genetic algorithm for multi-objective shop scheduling ; Zhu et al. [2] developed a hybrid differential-genetic algorithm using grey entropy parallel correlation for fitness calculation; Yue et al. [3] proposed FWGWO, a hybrid of grey wolf optimizer and fireworks algorithm, to enhance global optimization and convergence; Deng et al. [4] presented HAOAPSO, combining composite opposition-based learning, arithmetic optimization algorithm (AOA), and particle swarm optimization (PSO) to overcome AOA's weak local search and PSO's local optimum tendency ; Li et al. [5] hybridized NSGA-II with MOPSO, incorporating logic mapping initialization and dynamic operator selection ; Cui et al. [6] proposed MOPSO_TA with dual filing mechanisms, genetic operators, and parameter tuning, validated on 21 benchmarks; Lu et al. [7] developed GA-MBO, fusing genetic algorithm's global search and migratory bird optimization's local search for AS/RS-hybrid flow shop scheduling .

To address the aforementioned issues, this article proposes a multi-strategy hybrid particle swarm optimization (MS-MOPSO) algorithm. It adopts a co-evolving search mode integrating particle swarm optimization and grey wolf optimizer, guided by a leading individual selection mechanism that combines curve similarity (based on average Fréchet distance) and Pareto dominance. Additionally, a variable neighborhood search with three structures enhances local search capability. The algorithm is compared with three other methods using multi-objective DTLZ (1-5) test cases.

2. Realization of particle swarm optimization algorithm combining multiple strategies

2.1. General

The particle swarm optimization algorithm (PSO) is a classic swarm intelligent algorithm proposed by Kennedy et al. [8] in 1995. It solves optimization problems by imitating the cooperative mode in bird foraging behavior. The grey wolf optimizer algorithm (GWO) [9] simulates the class division structure and hunting mechanism of wolf swarm in the nature to achieve the optimizing process of the problem. The grey wolf optimizer algorithm is characterized in good global search, few control parameters and good robustness, has attracted the attention of many scholars in recent years and been applied to solve many practical optimization problems [10]. Sulaiman et al. [11] applied GWO to power system optimal reactive power scheduling, achieving lower power loss and voltage deviation. Gaafary et al. [12] used GWO to optimize static synchronous compensator allocation in power systems, minimizing load bus voltage deviation and system power loss. Pathak et al. [13] proposed a Levy flight-guided GWO for feature selection in image steganalysis, demonstrating desirable convergence and classification accuracy.

Bian et al. [14] developed a sulfur solubility prediction model for supercritical sulfur-containing gas by combining GWO with support vector machines, validating its effectiveness. Wang et al. [15] proposed a multi-mixed-feature GWO-based improved pulse-coupled neural network for multimodal medical image segmentation, which outperformed comparative algorithms in both subjective and objective evaluations. Jagatheesan et al. [16] optimized PID controller gains via GWO in a three-area grid-connected power network, yielding superior time-domain response (shorter stabilization time, lower overshoot) than other methods.

As a new swarm intelligent algorithm, GWO has the advantages of fast convergence speed, strong local search ability, etc. [17] and can complement the particle swarm optimization algorithm well to achieve better universality via complementary advantages.

2.2. Co-evolutionary swarm strategy

Traditional optimization algorithms adopt single-swarm evolutionary search, where uniform search and learning modes guide particles to converge in the same direction. This leads to reduced search capability within the feasible objective space during iteration, causing premature convergence and suboptimal results [18]. Thus, multi-swarm, multi-search-mode algorithms have emerged as a key research direction in recent years.

Current multi-swarm research mainly focuses on tuning design parameters for a single algorithm. When integrating two distinct algorithms as swarm search modes, their complementarity and inter-swarm information exchange must be considered. Particle Swarm Optimization (PSO) offers strong global optimization and simplicity, while the Grey Wolf Optimizer (GWO) excels in local search and convergence speed. Their combination leverages complementary advantages, enhancing feasible space search ability, generating more differentiated non-inferior solution sets, and providing superior candidate guiding individuals to improve search effectiveness.

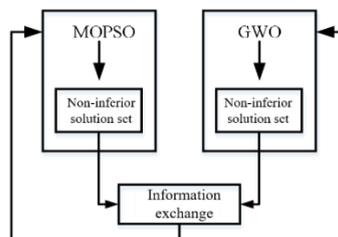


Fig. 1. Collaborative search of swarm

For inter-algorithm information interactions, shown in Fig. 1, an external archive stores non-inferior solutions from both algorithms. Guiding individuals for each algorithm are selected from this archive, enabling information exchange and adjusting respective search modes to achieve better search directions.

2.3 Mechanism for selecting leading individual

Similarly, the PSO and GWO algorithms similarly rely on leaders or swarms, so an effective leader selection mechanism is crucial for enhancing algorithm search performance. For the multi-objective particle swarm optimization algorithm (MOPSO), Pareto dominance is applied to select individuals and generate a non-inferior set. Leading individuals are then randomly chosen via roulette based on the adaptive values of individual grid density. This mechanism effectively maintains favorable distribution characteristics of the non-inferior solution set within the feasible search space. However, as the search scale expands, the number of non-inferior solutions continues to grow, leading to limitations of this random selection mechanism that hinder further improvement of algorithm search effectiveness. To address this issue, a selection mechanism is proposed for choosing leading individuals of PSO and GWO swarms. This mechanism is based on the discrete average Fréchet distance curve similarity matching principle and the Pareto dominance relationship. It enables the acquisition of Pareto solutions while ensuring the algorithm's search capability. The specific implementation is as follows:

Selection mechanism based on discrete average Fréchet distance and Pareto dominance

Start

Step 1: Calculate the objective function value for each individual generated by the algorithm;

Step 2: Use the Pareto domination relationship to select the non-inferior solution set and put it into the external archive set *Rep*;

Step 3: Take the minimum value of each objective vector in *Rep* and construct it as the reference point F_0^* .

Step 4: Use *z-score* normalization and *MAX* value normalization to map the reference point F_0^* and each individual objective function value F_g to the reference function value curve S_0^* and the curve S_g of each individual in the *Rep* set;

Step 5: Use the average Fréchet distance method to calculate the average Fréchet distance D_{aF} between the curves S_0^* and S_g , and take it as the individual fitness value;

Step 6: Sort the non-inferior solutions in *Rep* according to the D_{aF} value, and use them for selecting the global optimum of PSO and α , β and δ wolves of GWO

End

2.4 Local search

During the iterative optimization of the algorithm, the local search ability may have a great impact on the optimization results of the algorithm, and the

neighborhood structure determines the optimization ability of the local search algorithm [19]. The neighborhood structure is a mechanism, of which a small perturbation is given to the given solution to obtain another solution. For the variable neighborhood structure, the neighborhood structure in variable neighborhood search is systematically changed so that the algorithm has a higher probability of obtaining a global optimal solution than the search of a single neighborhood structure. Common neighborhood structures include the swap neighborhood search operator SWAP, the inverse neighborhood search operator INVERSE, and the insert neighborhood search operator INSERT as follows:

SWAP structure: In the individual coding, two swap points are randomly generated and their positions are swapped, as in Fig. 2.

INSERT structure: Two swap points are randomly selected and the latter point is inserted into the coding position behind the former one, as in Fig. 3.

INVERSE structure: Two swap points are randomly selected and their encoding method is reversed, as in Fig. 4.

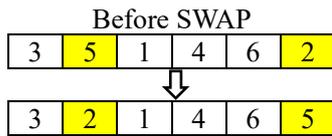


Fig. 2. SWAP neighborhood structure

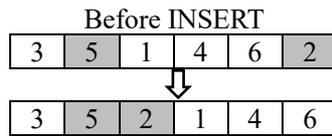


Fig. 3. INSERT neighborhood structure

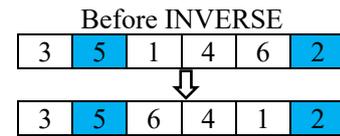


Fig. 4. INVERSE neighborhood structure

2.5 Process of MS-MOPSO algorithm

- (1) Set the specific parameters of the algorithm: Max iteration $Maxgen$, swarm size N , capacity of external archive set Nom , and replacement times of local search mode Ln .
- (2) Initialize the swarm, randomly generate the initial swarm $POP1$, and copy $POP1$ to generate swarm $POP2$.
- (3) Calculate the individual objective function values of $POP1$ and $POP2$ swarms and use the dominance relationship to select the non-inferior solutions of both swarms and put them into the same solution set NP .
- (4) Delete the dominance individuals in NP and put remaining individuals into in the external archive set Rep .
- (5) Adopt the selection mechanism based on average Frechet to select the global optimal particles of the particle swarm in the external archive Rep and α , β and δ wolves in the grey wolf search.
- (6) Iterative operation
 - 1) For individuals in $POP1$ and $POP2$, two mechanisms, PSO and GWO, are used as search modes to generate sub-swarms and calculate the corresponding objective function values of their individuals.

- 2) When the replacement time of the local search mode is not reached, the neighborhood structures of three local searches are probabilistically selected for local search of each individual. If dominating the original one, the new individual found may replace the original one. If both individuals do not dominate each other, the replacement is determined with a probability of 1/2.
 - 3) Follow the dominant relationship to select the non-inferior solution sets $P1$ and $P2$ among the offspring individuals and use the external archiving mechanism mentioned above to selectively place the individuals in $P1$ and $P2$ into the archive set Rep .
 - 4) Follow the selection mechanism based on average Frechet to select the globally optimal particles of the swarm in the external archive Rep and α , β and δ wolves in the grey wolf search.
 - 5) When the maximum number of iterations is not reached, repeat Step (1)-(4), or otherwise exit the iteration section.
- (7) The algorithm stops and the Pareto optimal set is output.

3. Multi-objective mode

In multi-objective test cases, most scholars currently use ZDT and DTLZ series. ZDT test cases involve the optimization of two goals, while DTLZ can be extended well as a high-dimensional multi-objective optimization because it can be extended to test optimization of multiple objectives. It is one of the most widely used test problems. The corresponding constraints on the function of M objectives in an n -dimensional DTLZ1 test case are described below [20].

Test case: DTLZ1

Optimization objective:

$$\begin{aligned}
 \min f_1(X) &= \frac{1}{2} x_1 x_2 \dots x_{M-1} (1 + g(X_M)), \\
 \min f_2(X) &= \frac{1}{2} x_1 x_2 \dots (1 - X_{M-1}) (1 + g(X_M)), \\
 \min f_{M-1}(X) &= \frac{1}{2} x_1 (1 - x_2) (1 + g(X_M)), \\
 \min f_M(X) &= \frac{1}{2} x_1 (1 - x_1) (1 + g(X_M)), \\
 &\text{s.t} \\
 &0 \leq x_i \leq 1 \quad (i = 1, 2, \dots, n) \\
 g(X_M) &= 100[|X_M| + \sum_{x_i \in X_M} ((x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5)))]
 \end{aligned}$$

Where, X_M is the set of the last k vectors in the n -dimensional vector, $k = (n - m + 1)$, and the $g(X_M)$ function needs to include $|X_M| = k$ variables. In this article, the test case with $M = 3$ of $n = 10$ dimensional vector objectives is

adopted. The DTLZ2–5 test cases used in this article are available in Reference [20].

4. Experiment and result analysis

In order to verify the performance of the MS-MOPSO algorithm, this article adopts DTLZ (1-5), which is commonly used in multi-objective testing, to perform validity testing of the algorithm. In order to reflect the optimization effect of the algorithm, the multi-objective particle swarm optimization algorithm (MOPSO), multi-objective grey wolf optimizer algorithm (MOGWO) and representative multi-objective evolutionary algorithm SPEA2 are used for comparison. In order to comprehensively evaluate the situation of the algorithms, the results of four algorithms are analyzed from three aspects: convergence index DM [21], diversity index SP [22] and coverage index C [18].

4.1 Parameter setting

In order to objectively reflect the effectiveness of the improved algorithm, the MS-MOPSO algorithm in this article is compared with MOPSO, MOGWO and SPEA2 algorithms, of which the parameter settings involved are as shown in Table 1.

Table 1

Parameter setting	
Algorithm	Parameter setting
SPEA2	$P_c = 0.7, P_m = 0.2, Group = 100, REP = 100$
MOPSO	$\omega = 0.5, \omega_{damp} = 0.5, C_1 = C_2 = 1, nGrid = 7, P_m = 0.1, Group = 100, REP = 100$
MOGWO	$nGrid = 10, Group = 100, REP = 100$
MS-MOPSO	$\omega = 0.6, C_1 = C_2 = 1, \omega_{damp} = 0.9, P_m = 0.2, nGrid = 8, Group = 100, REP = 100$

Among them, Group is a swarm, the number of iterations is $Maxit = 100$, REP is the number of external archive sets, ω is the inertia weight, ω_{damp} is the inertia weight attenuation rate, C_1 and C_2 are learning factors, and nGrid is the number of grids in each dimension.

4.2 Analysis of results

(1) The convergence index DM is used to measure how close the non-inferior solution set obtained by the algorithm is to the real Pareto front. The smaller the DM value is, the closer it is to the real Pareto front. From the DM as show in Fig. 5 to Fig. 9, it can be seen that four algorithms perform equivalently during the iteration process in the DTLZ (1, 5) test cases, but the algorithm in this article is optimal in converging to the real Pareto in DTLZ2 and DTLZ4 and its convergence in DTLZ3 is only next to SPEA2. In general, the MS-MOPSO algorithm in this article has certain advantages over other three algorithms in terms of fast

convergence, for the ideal adaptive point in its selection mechanism effectively guides the whole swarm.

(2) The diversity index SP is the distribution characteristic of the obtained objective values in the objective space and reflects the effectiveness of the algorithm's search mechanism. It can be seen from the distribution diagram in the Fig. 10 to Fig. 14 that the distribution uniformity of MS-MOPSO in this article is optimal in four test cases. The distribution uniformity of the Pareto optimal set searched is relatively better than other four algorithms but is not dominant absolutely in all cases. This phenomenon should be based on the guidance mode adopted in the selection mechanism of MS-MOPSO, while two algorithms of MOGWO and MOPSO are based on the mechanism of individual grid density adaptation. So, the probability of optimal solution distribution of two algorithms is relatively increased.

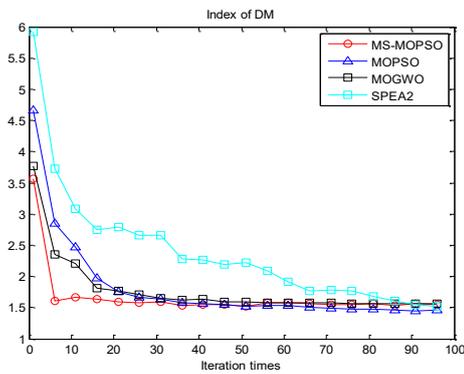


Fig. 5. DM Index of DTLZ1

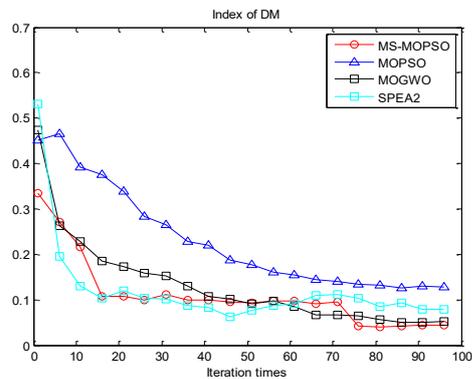


Fig. 6. DM Index of DTLZ2

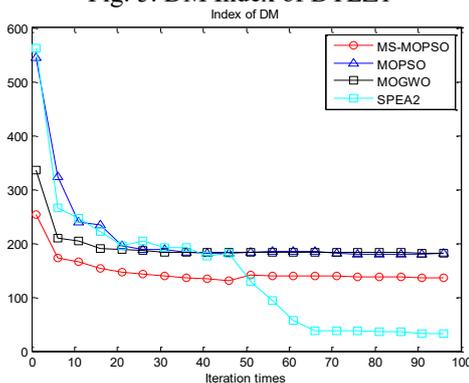


Fig. 7. DM Index of DTLZ3

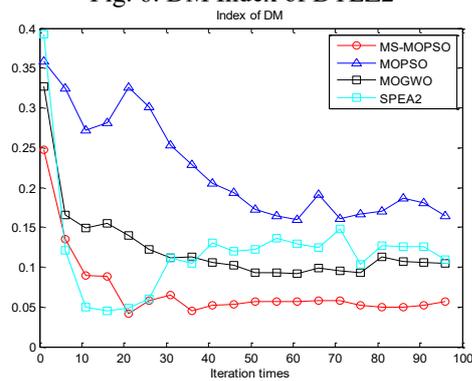


Fig. 8. DM Index of DTLZ4

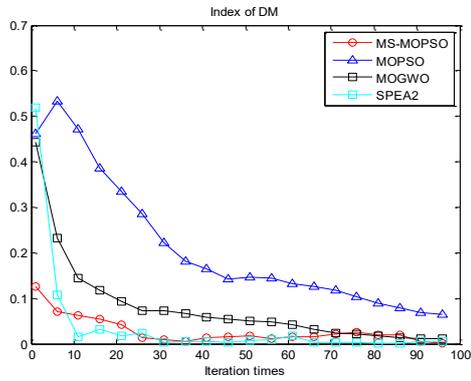


Fig. 9. DM Index of DTLZ5

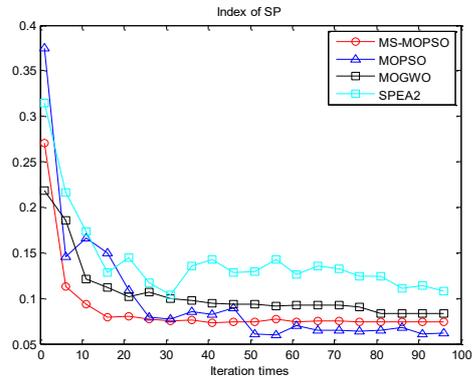


Fig. 10. SP Index of DTLZ1

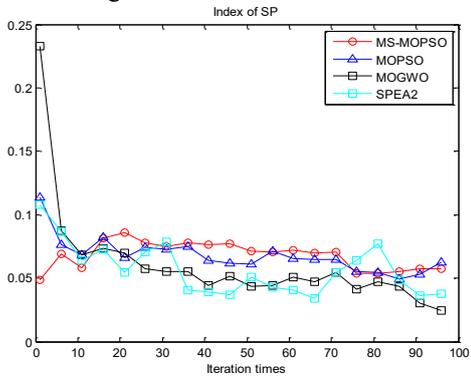


Fig. 11. SP Index of DTLZ2

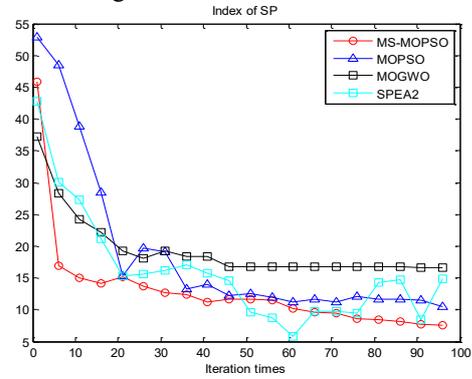


Fig. 12. SP Index of DTLZ3

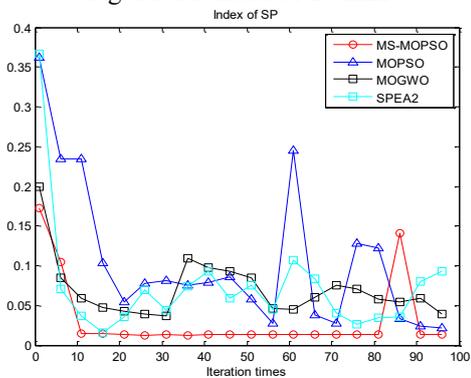


Fig. 13. SP Index of DTLZ4

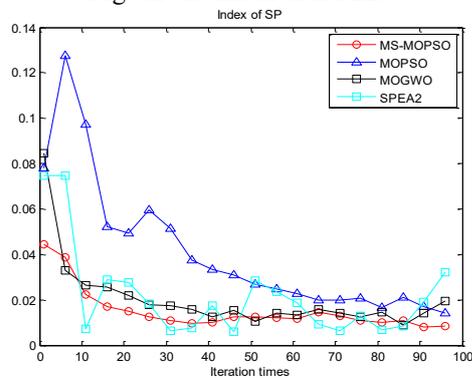


Fig. 14. SP Index of DTLZ5

(3) The coverage index C is used to evaluate the size of the relative coverage solution space between different Pareto solution sets generated by the algorithm. In terms of C index as show in Table 2, the MS-MOPSO algorithm in this article is dominant in the DTLZ (1-5) test cases while compared with MOGWO and MOPSO, the algorithm in this article is optimal in test cases while compared with the SPEA2 algorithm, but is inferior to the SPEA2 algorithm in two test cases, because the fast and strong dominant Pareto mechanism adopted by SPEA2 enables

it to find the true Pareto front quickly and well.

Table 2

Test function	C Index			
	C index			
DTLZ1	(MOPSO, MS-MOPSO)	0.13	(MS-MOPSO, MOPSO)	0.35
	(MOGWO, MS-MOPSO)	0.09	(MS-MOPSO, MOGWO)	0.44
	(SPEA2, MS-MOPSO)	0.44	(MS-MOPSO, SPEA2)	0.38
DTLZ2	(MOPSO, MS-MOPSO)	0.025	(MS-MOPSO, MOPSO)	0.73
	(MOGWO, MS-MOPSO)	0	(MS-MOPSO, MOGWO)	0.063
	(SPEA2, MS-MOPSO)	0.013	(MS-MOPSO, SPEA2)	0.55
DTLZ3	(MOPSO, MS-MOPSO)	0.05	(MS-MOPSO, MOPSO)	0.56
	(MOGWO, MS-MOPSO)	0	(MS-MOPSO, MOGWO)	0.98
	(SPEA2, MS-MOPSO)	0.98	(MS-MOPSO, SPEA2)	0.43
DTLZ4	(MOPSO, MS-MOPSO)	0	(MS-MOPSO, MOPSO)	0.28
	(MOGWO, MS-MOPSO)	0	(MS-MOPSO, MOGWO)	1
	(SPEA2, MS-MOPSO)	0	(MS-MOPSO, SPEA2)	0.51
DTLZ5	(MOPSO, MS-MOPSO)	0	(MS-MOPSO, MOPSO)	0.94
	(MOGWO, MS-MOPSO)	0.08	(MS-MOPSO, MOGWO)	0.42
	(SPEA2, MS-MOPSO)	0.07	(MS-MOPSO, SPEA2)	0.29

4.3 Significance test

MS-MOPSO yielded favorable evaluation metrics in DTLZ benchmark tests, yet its performance superiority over comparative algorithms cannot be definitively established without significance testing. Thus, the representative DTLZ4 test suite was selected for analysis, with the convergence index DM as the performance metric; the Kruskal-Wallis test—a non-parametric method superior for unknown or non-normal data distributions, which detects significant differences among multiple independent samples—was applied to statistically analyze the optimization results of SPEA2, MOPSO, MOGWO and MS-MOPSO. Each algorithm was run 40 times on DTLZ4, with DM values recorded for each run, yielding 160 samples for analysis. The Kruskal-Wallis test results (Table 3) show a test statistic of 21.8 and a corresponding p-value of 0.001. Since the p-value is far below the 0.05 significance level, the Null hypothesis can be rejected, indicating that the performance of the four algorithms on the DM metric is significantly different.

Table 3

Kruskal-Wallis test results					
Source	SS	df	MS	Chi-sq	Prob>Chi-sq
Groups	130976	3	65488	21.8	0.001
Error	1473070	156	5558.8		
Total	1604047	159			

In order to further compare the advantages and disadvantages of each algorithm, it is necessary to calculate the rank mean of the four algorithms. As

shown in Table 4. It can be seen from the table that the rank mean of MS-MOPSO is 109.9, which is significantly lower than that of the other three comparison algorithms.

Table 4

Rank average				
Index	MS-MOPSO	SPEA2	MOPSO	MOGWO
Estimated values of the mean ranks	109.9	152.8	173.2	137.3
Standard errors	7.3212	7.3212	7.3212	7.3212

Comprehensive analysis of the results searched by the algorithm in this article show that they are closer to the Pareto optimal set in the target space, and they have certain advantages in terms of convergence, distribution and optimization effect when dealing with high-dimensional multi-objective problems. The optimal algorithm is the optimization result of multiple search surfaces brought by multiple swarms, the ability of the selection mechanism to find the optimal aspect, and the reasonable local search ability.

5. Conclusion

To address multi-objective optimization, this article proposes MS-MOPSO that adopts two global search modes by leveraging the complementary features of the grey wolf optimizer and particle swarm optimization identified through comparative algorithm characteristic analysis, introduces a novel leading individual selection mechanism combining average Fréchet distance-based curve similarity matching and Pareto dominance to overcome limitations of traditional grid density-based random selection in large-scale leader solution set screening and guide efficient population search in the feasible target space, and integrates a variable neighborhood search with three structures to enhance local search capability and balance global-local optimization performance, these combined strategies expand the search scope and improve optimization effectiveness; simulation tests on the DTLZ1–5 multi-objective test suites, with comparisons against MOPSO, MOGWO and SPEA2 and performance analysis from coverage index C , diversity index SP and convergence index DM perspectives, verify the algorithm's effectiveness, further confirmed by significance testing on the four algorithms' solution sets.

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